Some problem-solving hints:

1. Don't panic.
2. If you're stuck, at least try *something*.
3. If you can't do something, don't.
4. If things gets weird, there's probably a mistake.
5. If you can't solve a problem, solve an easier problem first.
6. When in doubt, write it out.
7. Remember: $(a+b)^2 \neq a^2 + b^2$.

Laws of limits:

Suppose $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, and c is a constant. Then:

1. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$
4. $\lim_{x\to a}[f(x)g(x)] = (\lim_{x\to a} f(x)) \cdot (\lim_{x\to a} g(x))$
5. $\lim_{x\to a}[f(x)/g(x)] = (\lim_{x\to a} f(x)) \,/\, (\lim_{x\to a} g(x))$ if $\lim_{x\to a} g(x) \neq 0$
6. $\lim_{x\to a} c = c$
7. $\lim_{x\to a} x = a$

Problem 1. *(6 points) Simplify the following:*

a) $\log 5 + 5 \log 3$

b) $\sqrt{x^2}$

c) $\ln\left(\ln e^{e^5}\right)$

Problem 2. *(4 points) What is the domain of* $\log(x+2) + \log(x-2)$*?*

Problem 3. *(10 points) Compute* $\lim_{x\to 2} \dfrac{2x+1}{x+3}$ *using the Limit Laws (no shortcuts!). Use only one limit law in each step and tell me which law you are using.*

Problem 4. *(10 points) Suppose the maximum speed permitted on a highway is 65 mi/h. Suppose also that the fine for violating the speed limit is \$15 for every mile per hour past the limit.*

a) Express the amount of the fine f as a function of the driving speed x.

b) What is $f'(x)$?

c) What is the significance of $f'(x)$ in this situation?

Problem 5. *(5 points) Graph* $y = \tan(x + \pi/4)$.

Problem 6. *(5 points) Graph* $y = x^2 + 2x + 2$ *by completing the square.*

Problem 7. *(15 points) Let* $f(x) = \dfrac{(x+1)^2}{x^2}$.

a) Compute all vertical or horizontal asymptotes for f.

b) Graph $f(x)$ *using the previous answer.*

Problem 8. *(15 points) Let* $f(x) = \frac{x^2+3x+2}{x+2}$.

a) Compute $\lim_{x\to -2} f(x)$.

b) What does it mean for a function to be continous at a point a?

c) At what points is f continuous?

d) Graph $f(x)$.

Problem 9. *(10 points) Let* $f(x) = \begin{cases} x^2 \cos(1/x) + x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

a) Is $f(x)$ *continuous at 0? Prove that you are correct.*

b) Does $f'(0)$ *exist? If it does not exist, explain why not. If it does exist, what is its value?*

Problem 10. *(10 points) Use the limit definition of derivative to compute* $\frac{d}{dt}\sqrt{9-t}$. *Do NOT use any shortcuts.*

Problem 11. *(10 points) Does there exist a number* a *so that* $a^2 = 2^a$*? Your explanation should be very precise and clear (use full sentences).*