Some problem-solving hints:

1. Don't panic.
2. If you're stuck, at least try *something*.
3. If you can't do something, don't.
4. If things gets weird, there's probably a mistake.
5. If you can't solve a problem, solve an easier problem first.
6. When in doubt, write it out.
7. Remember: $(a + b)^2 \neq a^2 + b^2$.

Laws of limits:

Suppose $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, and $c$ is a constant. Then:

1. $\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. $\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. $\lim_{x \to a}[cf(x)] = c \lim_{x \to a} f(x)$
4. $\lim_{x \to a}[f(x)g(x)] = (\lim_{x \to a} f(x)) \cdot (\lim_{x \to a} g(x))$
5. $\lim_{x \to a}[f(x)/g(x)] = (\lim_{x \to a} f(x)) / (\lim_{x \to a} g(x))$ if $\lim_{x \to a} g(x) \neq 0$
6. $\lim_{x \to a} c = c$
7. $\lim_{x \to a} x = a$

**Problem 1.** *(6 points) Simplify the following:*

a) $\dfrac{(4^3 \cdot 5^{-2})^2}{2^6 \cdot 7}$

b) $\sqrt{x^2}$

c) $e^{2\ln 5}$

**Problem 2.** *(4 points) What is the domain of $\sqrt{|x|}$?*

**Problem 3.** *(10 points) Compute* $\lim_{x \to 2} \dfrac{x^2}{x + 3}$ *using the Limit Laws (no short-cuts!). Use only one limit law in each step and tell me which law you are using.*

**Problem 4.** *(10 points) Suppose we have the following income tax system: for the first $10,000 there is no tax. Any income past $10,000 is taxed at 10%.*

a) *Write the amount of tax paid as a function $f(x)$ of the income.*

b) *What is $f'(x)$?*

c) *What is the significance of $f'(x)$ in this situation?*

**Problem 5.** *(5 points) Graph* $y = \cos(x + \pi/4)$.

**Problem 6.** *(5 points) Graph* $y = x^2 + 4x + 1$ *by completing the square.*

**Problem 7.** *(15 points) Let $f(x) = \dfrac{(x-1)^2}{x^2}$.*

*a) Compute all vertical or horizontal asymptotes for $f$.*

*b) Graph $f(x)$ using the previous answer.*

**Problem 8.** *(15 points) Let $f(x) = \frac{x^2+3x-2}{x-2}$.*

*a) Compute $\lim_{x \to 2} f(x)$.*

*b) What does it mean for a function to be continous at a point a?*

*c) At what points is f continuous?*

*d) Graph $f(x)$.*

**Problem 9.** *(15 points) Let* $f(x) = \begin{cases} x^2 \sin(1/x) + x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

*a) Is $f(x)$ continuous at 0? Prove that you are correct.*

*b) Does $f'(0)$ exist? If it does not exist, explain why not. If it does exist, what is its value?*

**Problem 10.** *(10 points) Use the limit definition of derivative to compute $\frac{d}{dt}(5t - 9t^2)$. Do NOT use any shortcuts.*

**Problem 11.** *(10 points) Does there exist a number whose sine is equal to its square root? Your explanation should be very precise and clear (use full sentences).*