Problem 1. *Newton's Law of Cooling states that a hot object loses temperature faster than a warm one. More specifically, if T is the object's temperature and T_s is the surrounding temperature,*

$$\frac{dT}{dt} = k(T - T_s), \tag{1}$$

where k is some parameter that depends on what the object is made of.

Suppose we have a cup of coffee that starts off at 100C and takes 10 minutes to cool down to 70C. When will we be able to drink the coffee?

According to Brown and Diller 2008, "the optimal drinking temperature [for coffee is] approximately 136 degrees F (57.8 degrees C). "

(a) What information did I not provide? What would be a good value for it?

(b) Solve (1) for T. Hint: the function $y(x) = ce^x$ satisfies $y' = y$. What functions satisfy $y' = ky$? Take $z = T - T_s$ to simplify (1).

(c) What is the value of k?

(d) Is the coffee cold enough yet? (How long has it been since you started the problem?)

Problem 2. *You are inflating a balloon at one liter/minute.*

(a) Suppose the balloon is spherical. How fast is its radius changing after two minutes?

(b) Suppose the balloon is spherical. How fast is its surface area changing after two minutes?

(c) Suppose the balloon is a party balloon (a tube with radius 1 inch). How fast is its surface area changing after two minutes?