

Problem 1. Sketch the graph of a function that satisfies all of the conditions.

(a) Vertical asymptote $x = 0$,

$$\begin{array}{ll} f'(x) > 0 \text{ if } x < -2, & f'(x) < 0 \text{ if } x > -2 \text{ (} x \neq 0 \text{)}, \\ f''(x) < 0 \text{ if } x < 0, & f''(x) > 0 \text{ if } x > 0. \end{array}$$

(b) $f'(0) = f'(2) = f'(4) = 0$

$$\begin{array}{ll} f'(x) > 0 \text{ if } x < 0 \text{ or } 2 < x < 4, & \\ f'(x) < 0 \text{ if } 0 < x < 2 \text{ or } x > 4, & \\ f''(x) > 0 \text{ if } 1 < x < 3, & f''(x) < 0 \text{ if } x < 1 \text{ or } x > 3. \end{array}$$

Problem 2. Consider $f(x) = (x + 1)^5 - 5x - 2$.

- (a) Find the intervals of increase and decrease.
- (b) Find the local maximum and minimum values.
- (c) Find the intervals of concavity and the inflection points.
- (d) Use this information to sketch the graph of f .

Problem 3. Use the method in Problem 2 to graph $f(x) = \ln(x^4 + 27)$.