

From Wikipedia: “Although implicit in the development of calculus of the 17th and 18th centuries (Newton published his *Principia Mathematica* in 1687), the modern idea of the limit of a function goes back to Bolzano who, in 1817, introduced the basics of the  $\epsilon$ - $\delta$  technique to define continuous functions. However, his work was not known during his lifetime. Cauchy discussed limits in his *Cours d’analyse* (1821) and gave essentially the modern definition, but this is not often recognized because he only gave a verbal definition. Weierstrass first introduced the  $\epsilon$ - $\delta$  definition of limit in the form it is usually written today. He also introduced the notations  $\lim$  and  $\lim_{x \rightarrow x_0}$ . The modern notation of placing the arrow below  $\lim$  is due to Hardy in *A Course of Pure Mathematics* in 1908.”

**Problem 1.** *How many years did it take for mathematicians to come up with the Limit Laws as we see them? How many mathematicians did it take?*

**Problem 2.** *Write down the 11 Limit Laws. Which of the rules allow us to “plug the point in”?*

**Problem 3.** *Compute the limit, showing **all** work and stating which Limit Law you used (think of the expression as a single piece that you can only change using the Limit Laws):*

$$\lim_{x \rightarrow -1} \sqrt[5]{\frac{2x+3}{x^2}}$$

**Problem 4.** Compute the following limits, showing some work. Discuss which laws or theorems you are using.

(a)  $\lim_{x \rightarrow 3} (5x^3 - 3x^2 + x - 6)$

(b)  $\lim_{x \rightarrow -2} \sqrt{u^4 + 3u + 6}$

(c)  $\lim_{x \rightarrow 3} (2x + |x - 3|)$

(d)  $\lim_{x \rightarrow -2} \frac{x + 2}{x^3 + 8}$

(e)  $\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$

(f)  $\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin(\pi/x)$

(g)  $\lim_{x \rightarrow 1} g(x)$ , if you know that  $2x \leq g(x) \leq x^4 - x^2 + 2$  for all  $x$ .

(h)  $\lim_{x \rightarrow 1} h(x)$ , if you know that  $2x \leq h(x) \leq x^4 - x^2 + 2$  for all  $x > 1$ .