5 pages. 6 problems. 100 points. No calculators. Show all work.

Problem 1 (5 points each). *Suppose you have the following sets:*

$$A = \{\tau, blue, \tau, cat\} \qquad B = \{\tau, cat, blue\} \qquad C = \{A, \emptyset, B\}$$

(a) How many elements does B have? List them without repetition.

(b) How many elements does C have? List them without repetition.

(c) Is B a subset of A? Why or why not?

(d) Which (if any) of the three sets contain the empty set as an element?

Problem 2 (5 points each). *You have a standard deck of cards.*

(a) You shuffle the deck. What is the probability that the bottom card is a king?

(b) What is the probability that the bottom two cards are the same suit as the top one?

Problem 3 (5 points each). *Consider the following set of names:*

$D = \{$*Walker, Robena, Sebastian, Dorian, Bobby, Lyndi, Lilly, Sandy, Jordana, Tiffany*$\}$

(a) *Use one Venn diagram to illustrate the following sets (write out all the names in the relevant parts of the Venn diagram):*

E = *the set of names in* D *that end in* y *(4 of them),*

F = *the set of names in* D *that contain an* n *(7 of them).*

(b) *Suppose you pick a name randomly from* D. *What is the probability it ends in a* y*?*

(c) *Suppose you pick names randomly from* D *until you get one that contains an* n. *What is the probability that it also ends in a* y*?*

(d) *How many ways are there to choose three different names from* D *if the order doesn't matter?*

Problem 4 (5 points each). *Consider the following games:*

A: You flip a coin. If it's heads, you get 4 points. If it's tails, you get 6 points.

B: You roll two dice. If they're the same, you lose 6 points. Otherwise, you gain 6 points.

(a) Compute the expected value for game A.

(b) Compute the expected value for game B.

(c) Compute the variance each game.

(d) If you're feeling lucky, which game should you play to get more points? Why?

Problem 5 (5 points each). *Your students get the following grades on a quiz (out of 10 points):*

$$7, 7, 9, 5, 9, 10, 8, 10, 6, 8, 6, 5, 8, 9, 6, 7, 10, 5, 7, 9, 4, 6$$

(a) What are the maximum, minimum, and median scores?

(b) Use a bar graph to display the scores.

(c) Use a pie chart to show how many students get a 9 or better.

Problem 6 (20 points total: 5, 10, 5)**.** *Columbia University has 1,500 incoming students each year, and a 4-year graduation rate of 90%. Suppose you have developed a test that predicts whether a student will leave early. Suppose your test has a 1% false positive rate, and an 20% false negative rate (here, a "positive" means the student will **not** graduate).*

(a) How many of the incoming students will graduate? How many won't?

(b) If you test all the students, how many will you decide are not going to graduate?

(c) If you decide a specific student will not graduate, what are the chances that the student actually won't graduate?