

Problem 1. Here's the Washington, DC, metro system. Think of it as a graph whose vertices are the stations, with edges between stations that are connected by at least one rail line.



- (a) What valence do most stations have?
- (b) How many stations have valence 3?
- (c) Which stations have valence 5?
- (d) Starting at the Pentagon station (just left of the river), how many stations can you get in at most 5 stops?
- (e) What is the shortest cycle in this graph?

Problem 2. *Here are some Western states:*

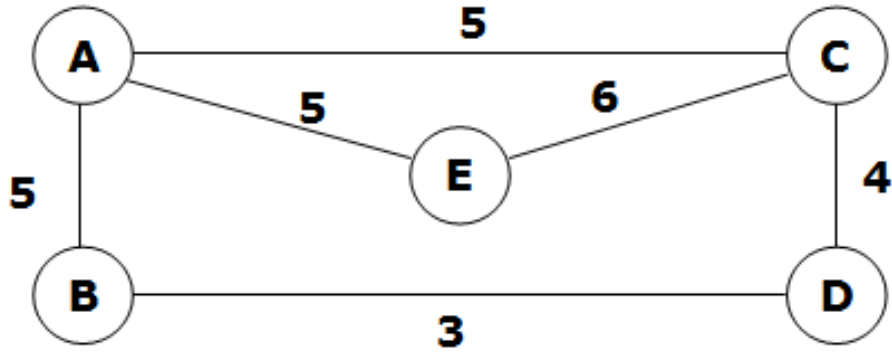


(a) *Draw the adjacency graph for this map. That is, replace each state with a vertex, and connect the vertices if the corresponding states touch.*

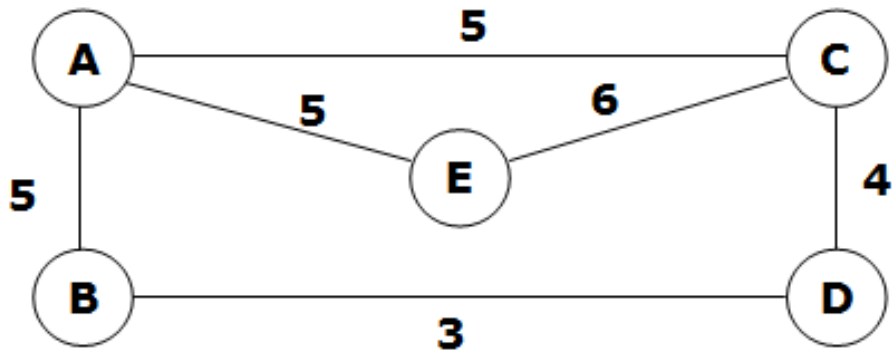
(b) *Compute the valence of each vertex in your graph. What does it mean in terms of the map?*

(c) *Is it possible to start in some state, cross each border exactly once, and return home? If so, draw your path on the map. If not, convince me you're right.*

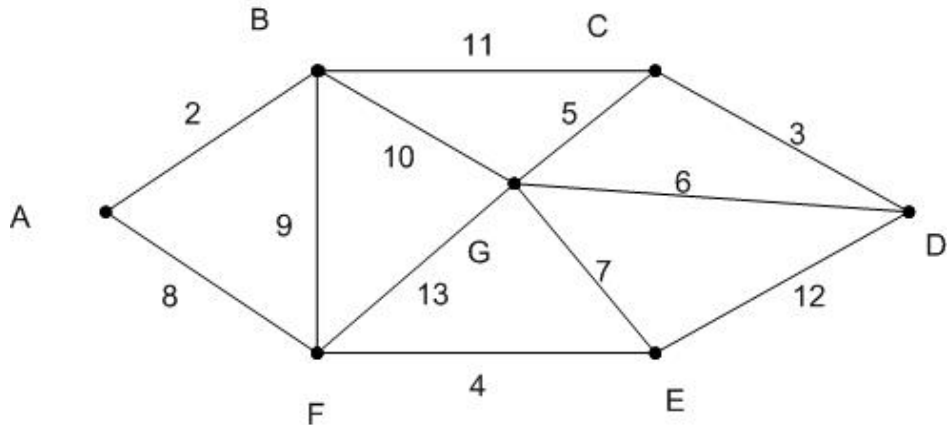
Problem 3. Use Dijkstra's algorithm to find the distance from A to D:



Problem 4. Use Dijkstra's algorithm to find the distance from B to C:



Problem 5. Use Dijkstra's algorithm to find the distance from B to E:



Problem 6. *Think of the “friends graph” for people in our class, where people become vertices, and there’s an edge between two people if they are friends.*

(a) Is the friends graph connected?

(b) What does having a really high valence mean in terms of people?

Problem 7. *Google’s PageRank algorithm analyzes the significance of web pages by looking at how they are connected to each other. If we want to make a graph whose vertices are web pages, what should the edges be?*

Problem 8. *How many edges are there in a complete graph with 6 vertices?*