

# Math 675 Homework 1

Due 8/29/2018

Responses must be typed, with a two-page maximum and LaTeX highly recommended. Grade will be based on style (2 points) and the best three problems (2 points each), but all five are highly recommended.

1. Let  $\{v_1, \dots, v_m\} \subset \mathbb{C}^m$  be nonzero vectors such that  $(v_i, v_j) = 0$  when  $i \neq j$ . Show that the set  $\{v_1, \dots, v_m\}$  is linearly independent.
2. Let  $\{v_1, \dots, v_m\} \subset \mathbb{C}^m$  be a set of linearly independent vectors, and suppose that  $w \in \mathbb{C}^m \setminus \text{span}\{v_1, \dots, v_m\}$ . Prove that the set  $\{v_1, \dots, v_m, w\}$  is linearly independent.
3. Let  $U, V \subset \mathbb{C}^m$  be two subspaces. Prove that

$$\dim(U + V) + \dim(U \cap V) = \dim U + \dim V.$$

4. Let  $A$  be an  $m \times n$  matrix. Show that the null space and range of  $A$  satisfy

$$\dim N(A) + \dim R(A) = n.$$

5. Let  $A$  be an  $m \times n$  matrix. Show that

$$R(A)^\perp = N(A^*).$$