Math 675 Homework 10
Due 11/14/2018

1. Prove that the functional $F(f) = \int_a^b f(t)\cos(t)dt$ is a continuous functional on $C_\infty[a, b]$. What could we replace $\cos(t)$ with?
2. Give an example of an inner product space V and an orthonormal system $\{e_i\}$ such that V contains no non-zero element orthogonal to every e_i, even though $\{e_i\}$ does not span V.
3. Prove that both of the following are subspaces of ℓ^2:
   (a) The set of all (x_i) such that $x_1 = x_2$.
   (b) The set of all (x_i) such that $x_k = 0$ for all even k.
4. (Optional) Prove that every nested sequence of nonempty closed bounded convex sets in a Hilbert space (of any dimension) has a nonempty intersection. (Do not use anything beyond Chapter 16.)