

Math 675 Homework 2

Due 9/05/2018

Responses must be typed, with a two-page maximum and LaTeX highly recommended. Grade will be based on style (2 points) and all of the problems.

- (2 points) Prove the Cauchy-Schwartz inequality, following Problem 2 on Page 45.
- (2 points each) Read through the proof that, for $p \in [1, \infty)$, d_p is a metric on \mathbb{R}^n (Example 10 starting on Page 41).
 - Explain why the proof fails for $p \in (0, 1)$.
 - Use an example to show that $d_{1/2}$ is not a metric in \mathbb{R}^2 .
- (2 points each) Show that in \mathbb{R}^n the metric d_∞ is the limit of the functions d_p as $p \rightarrow \infty$:
 - Experimentally by drawing a sequence of unit balls in \mathbb{R}^2 . (Hint: use Wolfram Alpha or other software, and then include the graphics into your write-up.)
 - Provide a proof that

$$\max |x_i - y_i| = \lim_{p \rightarrow \infty} \left(\sum_{i=1}^n |x_i - y_i|^p \right)^{1/p} .$$

- (2 points) Find an isometry (see Definition 2 on p.44) between $C_\infty[a, b]$ and $C_\infty[c, d]$ for any $a < b$ and $c < d$.