Math 675 Homework 3

Due 9/12/2018

Responses must be typed, with a two-page maximum and LaTeX highly recommended. Grade will be based on style (2 points) and all of the problems.

Let $A \subset X$ be a subset of a metric space (X, d).

1. Prove that $x \in X$ is a limit point of A if and only if for every $\epsilon > 0$ the ball $B_\epsilon(x)$ contains a point of A not equal to x.
2. Prove that the closure of A equals the union of the set of isolated points A with the set of limit points of A.
3. Let $a < b \in \mathbb{R}$. Prove that the closed interval $[a, b]$ is also closed in the topological sense of the word.
4. What is the closure of $\mathbb{Q} \subset \mathbb{R}$? Prove that you are correct.
5. Look up Lipschitz functions, and notice that definition of bi-Lipschitz is a generalization of the Lipschitz condition. Another common condition in real analysis is the Hölder condition. This leads to a notion of equivalence that is intermediate between bi-Lipschitz equivalence and homeomorphic equivalence.
   (a) Define bi-Hölder equivalence between metric spaces, analogously to the definition of bi-Lipschitz equivalence.
   (b) Suppose X and Y are bi-Lipschitz equivalent. Prove that they are also bi-Hölder equivalent.
   (c) Suppose X and Y are bi-Hölder equivalent. Prove that they are also homeomorphic.