

Math 675 Homework 4

Due 9/19/2018

Responses must be typed, with a two-page maximum and LaTeX highly recommended. Grade will be based on style (2 points) and all of the problems.

1. Prove that the following are equivalent for a function $f : X \rightarrow Y$ between two metric spaces:
 - (i) f is continuous,
 - (ii) $f^{-1}(A)$ is an open set for every open set $A \subset Y$,
 - (iii) $f^{-1}(B)$ is a closed set for every closed set $B \subset Y$.
2. Prove that:
 - (a) The set of invertible 2-by-2 matrices is open. (Interpret the space of all 2-by-2 matrices as \mathbb{R}^4 with the usual topology.)
 - (b) The set of determinant-one 2-by-2 matrices is closed.
3. Let A be a linearly independent set in a metric vector space V .
 - (a) Assuming that V is finite-dimensional, prove that the set $\mathbb{R}A = \{\lambda a : \lambda \in \mathbb{R}, a \in A\}$ is closed.
 - (b) Give an infinite-dimensional example for which $\mathbb{R}A$ is not closed.
4. Provide examples of:
 - (a) An infinite union of closed sets that is not closed.
 - (b) An infinite intersection of open sets that is not open.