

Math 675 Homework 7

Due 10/10/2018

1. Verify that the spaces $C[a, b]$, ℓ_2 , c , c_0 , m , and \mathbb{R}^∞ are infinite-dimensional.
2. Let f, f_1, \dots, f_n be linear functionals on a linear space L such that for all x one has that $f_1(x) = f_2(x) = \dots = f_n(x) = 0$ implies $f(x) = 0$. Prove that there are constants a_1, \dots, a_n such that $f = a_1 f_1 + \dots + a_n f_n$.
3. Let V be a vector space. Prove that the set of linear functionals on V is a vector space. It is commonly denoted V^* . Give an example of a non-trivial functional on V^* .
4. Explain the big-O and little-o notation for sequences in terms of quotient spaces.