16 pages, 23 questions, 200 points total. 3 hours.

Some problem-solving hints:

1. Don't panic.
2. If you're stuck, at least try *something*.
3. If you can't do something, don't.
4. If things gets weird, there's probably a mistake.
5. If you can't solve a problem, solve an easier problem first.
6. When in doubt, write it out.
7. Remember: $(a+b)^2 \neq a^2 + b^2$.
8. If a method doesn't help, admit it.
9. No work – no credit.

Good luck!

Problem 1 (2 points each). *Simplify:*

(a) $\log_5 125$

(b) $\log^{-1}(e^2)$

(c) $\sqrt{(-3)^2}$

Problem 2 (4 points). *Graph* $\cos(1+x)+2$.

Problem 3 (2 points each). *Simplify completely:*

(a) $b^8(2b)^4$

(b) $8^{4/3}$

(c) $\log_3 100 - \log_3 18 + \log_3 50$

Problem 4 (4 points). *State the domain and range of* $\cos^{-1}(e^x)$. *Explain.*

Problem 5 (10 points)**.** *Find the vertical and horizontal asymptotes of* $f(x) = \dfrac{2e^x}{e^x - 5}$.

Problem 6 (5 points each)**.** *Compute the following limits:*

(a) $\lim_{x\to 2} \dfrac{1+x}{(x+2)^2}$

(b) $\lim_{t\to 0} \dfrac{\sqrt{1+t}-\sqrt{1-t}}{t}$

(c) $\lim_{x\to\infty}(x+\sqrt{x^2+2x})$

Problem 7 (10 points). *Consider the function*

$$f(x) = \begin{cases} x^2 \cos(1/x) + x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(a) Is f continuous at 0? Why?

(b) Use the definition of derivative to compute $f'(0)$.

Problem 8 (2 points each). *Compute the following derivatives:*

(a) $\frac{d}{dx}(x^2 + 3x - 3)$

(b) $\frac{d}{dx}x^{\sqrt{x}}$

(c) $\frac{d}{dx}\dfrac{2x+1}{\sin(x)+1}$

Problem 9 (4 points). *Is it true that* $\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$*? Why or why not?*

Problem 10 (10 points). *Write down the tangent line to the curve* $x^2 = (y-1)^3$ *at the point* $(8,3)$.

Problem 11 (10 points). *Two people start walking from the same point. One walks east at* $3mi/h$ *and the other walks northeast at* $2mi/h$. *How fast is the distance between the people changing after 15 minutes?*

Problem 12 (10 points)**.** *Show that there is a number a such that $a^5 - a + 1 = 0$. Make a guess for the value of a, and use one step of Newton's method to improve your guess.*

Problem 13 (5 points)**.** *State the Extreme Value Theorem and draw a picture illustrating it.*

Problem 14 (10 points). *At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?*

Problem 15 (5 points). *Use the midpoint rule with $n = 3$ to approximate $\int_0^\pi \cos(x)dx$.*

Problem 16 (15 points). *Let* $f(x) = 2\sqrt{x} - x$.

(a) Where is f *increasing/decreasing?*

(b) Where is f *concave up/down?*

(c) Graph $f(x)$ *using the above information. Label critical points and inflection points.*

Problem 17 (5 points each). *Compute the following integrals:*

(a) $\int_0^2 (2x-3)(4x^2-1)dx$

(b) $\int_0^3 |2x-1|dx$

(c) $\int (4x^3-3x^2+2x)dx$

Problem 18 (10 points)**.** *Find the derivative of* $g(x) = \int\limits_{1-2s}^{1+2s} t \sin t dt$.

Problem 19 (5 points)**.** *Which is bigger,* $\int_0^1 e^{x^2} dx$ *or* $\int_0^1 e^{2x} dx$*? Why?*

Problem 20 (10 points)**.** *Sketch the region enclosed by the curves* $x = y^4, y = \sqrt{2-x}$, *and* $y = 0$, *and compute its area.*

Problem 21 (10 points)**.** *Find the average value of* $\sec^2(\theta/2)$ *on the interval* $[0, \pi/2]$.

Problem 22 (10 points). *The base of a solid S is a circle of radius 2. The cross-sections of S perpendicular to the x-axis are circles.*

(a) Sketch S including its base and cross-sections (and label the axes).

(b) Integrate to compute the volume of S.

Problem 23 (10 points). *The region R enclosed by $x = y^2 + 1$ and $x = 2$ is rotated around the line $y = -2$, producing a solid S.*

(a) Sketch R and S.

(b) Compute the volume of S.