Seven pages, seven questions, 75 points total.

Some problem-solving hints:

- 1. Don't panic.
- 2. If you're stuck, at least try something.
- 3. If you can't do something, don't.
- 4. If things gets weird, there's probably a mistake.
- 5. If you can't solve a problem, solve an easier problem first.
- 6. When in doubt, write it out.
- 7. Remember: $(a+b)^2 \neq a^2 + b^2$.
- 8. If a method doesn't help, admit it.

Problem 1 (5 points each). Compute the following derivatives, but don't simplify:

(a)
$$\frac{d}{dx} \frac{7x^3 + 2x - 7}{2x + 1}$$

(b)
$$\frac{d}{dx}\sin^2 x$$

(c)
$$\frac{d}{dx}(2x)^x$$

Problem 2 (10 points). Find the equation for the tangent line to the curve $x^2 + y^2 = 25$ at the point (3,4).

Problem 3 (10 points). Use linear approximation to estimate the value of

 $e^{0.03}$

Problem 4 (10 points). You are inflating a spherical balloon at a rate of $100 \text{cm}^3/\text{s}$. How fast is its radius changing when the volume is 1000cm^3 ? (Hint: $\frac{4}{3}\pi r^3$ might be relevant).

Problem 5 (10 points). Compute the limit

$$\lim_{x \to \infty} \frac{x}{e^x + 2}$$

Problem 6 (10 points). Suppose you drove from Urbana to Chicago (139 miles) in three hours. The speed limit is 65 miles/hour. Did you go over the speed limit? Explain.

Problem 7 (10 points). A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one fo the sides of the rectangle. How can he do this so as to minimize the cost of the fence?