

5 pages. 10 problems. 100 points. No calculators. Show all work.

**Problem 1** (5 points each).(a) Convert  $74_{10}$  to base-2.

$$74 = 64 + 10 = 64 + 8 + 2$$

$$2^6 \quad 2^3 \quad 2^1$$

$$74_{10} = 1001010_2$$

(b) Convert  $11010_2$  to base-10.

$$0 \times 1 + 1 \times 2^1 + 0 \times 2^2 + 1 \times 2^3 + 1 \times 2^4$$

$$2 + 8 + 16 = 26$$

$$\text{So } 11010_2 = 26_{10}$$

**Problem 2** (15 points). Most credit cards implement the Luhn Algorithm (patented in 1960), which works as follows (let's use the example 4485723586944236):

1. Remove the last digit, which is the checksum (6, leaving 448572358694423).
2. Double every other remaining digit (8, 4, 16, 5, 14, 2, 6, 5, 16, 6, 18, 4, 8, 2, 6).
3. Add up all the digits ( $8 + 4 + 1 + 6 + 5 + 1 + 4 + 2 + 6 + 5 + 1 + 6 + 6 + 1 + 8 + 4 + 8 + 2 + 6 = 84$ ).
4. Take the last digit of your sum (4) and subtract it from 10 (6). That is the checksum from 1.

Is 59240473 a valid credit card number? Why?

1. Leave 5924047 (checksum is 3).
2. Double: 10, 9, 4, 4, 0, 4, 14
3. Add:  $1 + 0 + 9 + 4 + 4 + 4 + 1 + 4 = 27$
4. Last digit is 7, and  $10 - 7 = 3$

~~Since  $9 \neq 3$ , this is not a valid credit card number; it doesn't fit the algorithm.~~

So the checksum should be 3, and it is 3.  
So the card is valid.

**Problem 3** (5 points each).

(a) Write out the multiplication table mod 8.

1	2	3	4	5	6	7	8
2	4	6	0	2	4	6	
3	6	1	4	7	2	5	
4	0	4	0	4	0	4	
5	2	7	4	1	6	3	
6	4	2	0	6	4	2	
7	6	5	4	3	2	1	
8							

(b) Which numbers don't have square roots?

~~2~~, ~~3~~, ~~5~~, ~~6~~, ~~7~~

(c) Which numbers are zero divisors?

2, 4, 6

(d) Find all solutions to  $2x = 4 \pmod{8}$ .

$x = 2, 6$

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Problem 1: 1 point. Given the set  $S$  of all letters after removing the set  $T$  of letters

$S = \{x \mid x \text{ is a letter and } x \notin T\}$

What is the answer?

Problem 2: 1 point. What are the truth values for  $A$ ,  $B$ , and  $C$ ?

A	B	C	A or B	(A or B) and C
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

$\rightarrow$  T F T  
 $\rightarrow$  F T T

Problem 3: 1 point. Find all the prime numbers up to 50.

<del>1</del>	2	3	<del>4</del>	5	6	7	<del>8</del>	<del>9</del>	10
11	<del>12</del>	13	<del>14</del>	<del>15</del>	16	17	18	19	20
<del>21</del>	22	23	<del>24</del>	<del>25</del>	26	27	28	29	30
31	32	33	<del>34</del>	<del>35</del>	36	37	38	39	40
41	42	43	<del>44</del>	<del>45</del>	46	47	48	49	50

2, 3, 5, 7, 11, 13, 17, 19, 23, 29,  
31, 37, 41, 43

**Problem 7** (5 points each). Let  $A$  be "Charlie likes pool and basketball, but not volleyball".

(a) What are the three pieces of  $A$ ? Call them  $B$ ,  $C$ , and  $D$ .

$B =$  "Charlie likes pool"  
 $C =$  "Charlie likes Basketball"  
 $D =$  "Charlie likes volleyball"

(b) Write  $A$  as a Boolean expression involving  $B$ ,  $C$ , and  $D$ .

$A = B \text{ and } C \text{ and } (\text{not } D)$

(c) Negate the Boolean expression from (b) and simplify.

$\text{not } (B \text{ and } C \text{ and } (\text{not } D))$   
 $(\text{not } B) \text{ or } (\text{not } C) \text{ or } D$

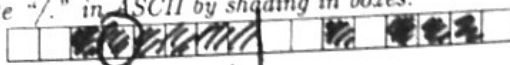
(d) Write out "not  $A$ " in words, based on your answer in part c.

Several options: Charlie does not like pool, or basketball, or he does like volleyball.  
Charlie likes volleyball, or dislikes pool or basketball.

### ASCII Chart

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SP	!	"	#	\$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{		}	~	DEL

Problem 8 (15 points). Write "." in ASCII by shading in boxes:



Show your work below:

$$/ \Rightarrow 2F = 2 \times 16 + 15 = 32 + 8 + 4 + 2 + 1$$

$$00101111_2$$

$$\bullet \Rightarrow 2E = 2 \times 16 + 14 = 32 + 8 + 4 + 2$$

$$00101110_2$$

Problem 9 (5 points). What is the difference between http and https?

https encrypts the data sent to / from a website.

Problem 10 (5 points). Answer in 20 words or less: what was your favorite Engineering Open House exhibit, and why?

The Oculus Rift demo (3D goggles), because 3D vision is cool.