

5 pages. 10 problems. 100 points. No calculators. Show all work.

**Problem 1** (5 points each).

(a) Convert  $74_{10}$  to base-2.

(b) Convert  $11010_2$  to base-10.

**Problem 2** (15 points). Most credit cards implement the Luhn Algorithm (patented in 1960), which works as follows (let's use the example  $4485723586944236$ ):

1. Remove the last digit, which is the checksum (6, leaving  $448572358694423$ ).
2. Double every other remaining digit ( $8,4,16,5,14,2,6,5,16,6,18,4,8,2,6$ ).
3. Add up all the digits ( $8 + 4 + 1 + 6 + 5 + 1 + 4 + 2 + 6 + 5 + 1 + 6 + 6 + 1 + 8 + 4 + 8 + 2 + 6 = 84$ ).
4. Take the last digit of your sum (4) and subtract it from 10 (6). That is the checksum from 1.

Is  $59240473$  a valid credit card number? Why?

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**Problem 3** (5 points each).

(a) *Write out the multiplication table mod 8.*

(b) *Which numbers don't have square roots?*

(c) *Which numbers are zero divisors?*

(d) *Find all solutions to  $2x = 4 \pmod{8}$ .*

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**Problem 4** (5 points). *Decode the shift cipher (after guessing the shift):*  
*UIF BOTXFS*

**Problem 5** (5 points). *Write out the truth table for “ $(A \text{ xor } B) \text{ and } C$ ”.*

**Problem 6** (5 points). *Find all the prime numbers up to 50.*

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**Problem 7** (5 points each). Let  $A$  be “Charlie likes pool and basketball, but not volleyball”.

(a) What are the three pieces of  $A$ ? Call them  $B$ ,  $C$ , and  $D$ .

(b) Write  $A$  as a Boolean expression involving  $B$ ,  $C$ , and  $D$ .

(c) Negate the Boolean expression from (b) and simplify.

(d) Write out “not  $A$ ” in words, based on your answer in part c.

