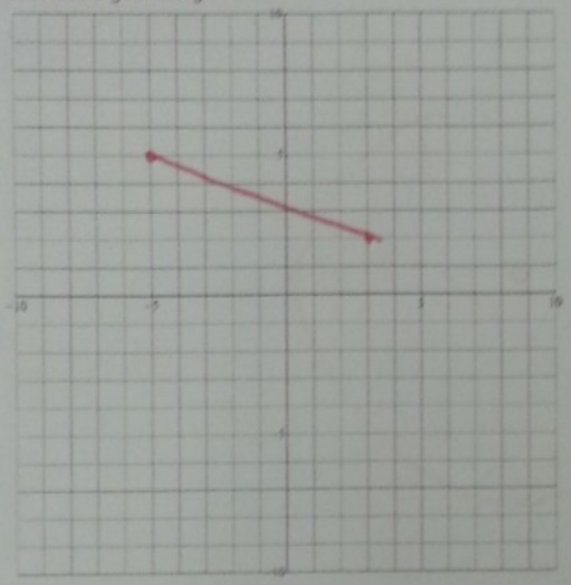
Math 181 F1 Exam 3 Version A Name:

6 pages. 8 problems. 100 points. No calculators. Show all work.

Problem 1 (10 points each). *In each geometry, draw a geodesic between the points $A = (-5, 5)$ and $B = (3, 2)$, and find the distance between the points.*

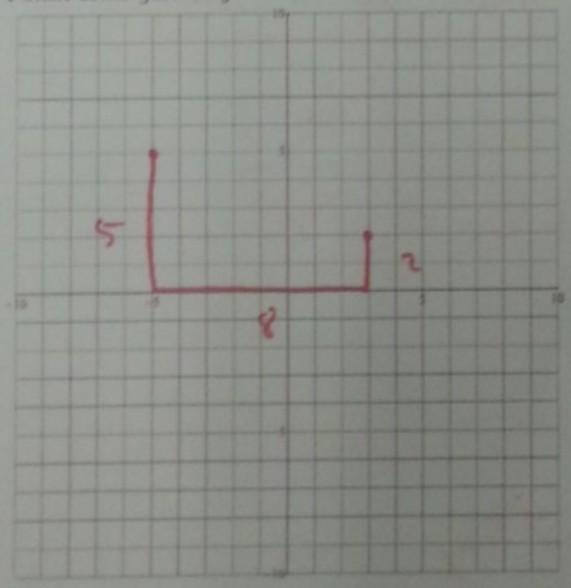
(a) Euclidean geometry



$\sqrt{(3+5)^2+(5-2)^2} = \sqrt{8^2+3^2}$

$= \sqrt{64+9} = \sqrt{73}$

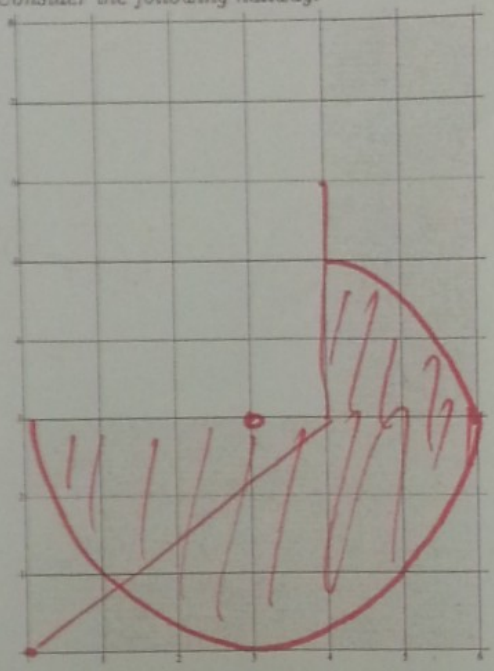
(b) Polish comb geometry



$5+8+2 = 15$

Points: 1pt
Geodesic: 2pts (1pt if extra lines)
Distance: 2 pts.

Problem 2 (5 points each). *Consider the following hallway:*



(a) Compute the hallway distance from the point $(0,0)$ *to the point* $(4,6)$.

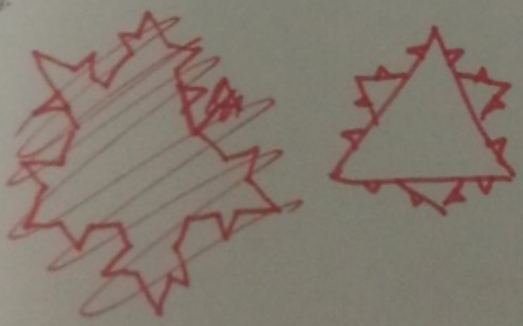
Diagonal to (4,3), then up to (4,6):

$\sqrt{4^2+3^2}=5$ 3

So $5+2=$ 8

(b) Draw on the picture a (hallway-geometry) disk of radius 3 *centered at the point* $(3,3)$.

Problem 3 (10 points). *Draw a Von Koch snowflake, with enough detail to convince me you know what you're doing.*



Problem 4 (5 points each). *Suppose you have a surface that has 4 vertices, 4 faces, and 6 edges.*

(a) *Find the Euler characteristic of the surface.*

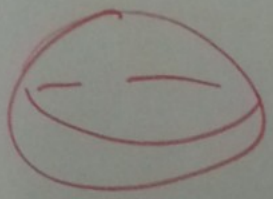
$V-E+F = 4-6+4 = 2$

(b) *Find the genus of the surface.*

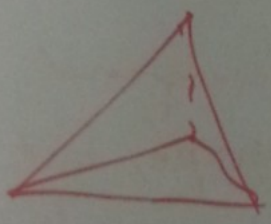
$2-2g = 2$

$g = 0$

(c) *Draw the surface in 3D just based on information about its genus (don't show the vertices, faces, and edges).*

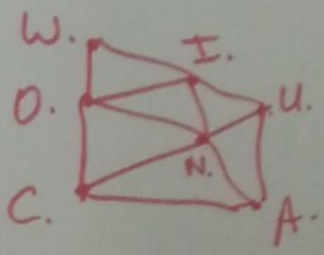


(d) *Draw the same surface in 3D in a way that shows the vertices, faces, and edges - and with flat faces.*



Problem 5 (5 points each). *Here are some Western states:*

(a) Draw below the adjacency graph for this map. That is, replace each state with a vertex, and connect the vertices if the corresponding states touch.



(b) What is the valence of the vertex corresponding to Nevada? What does the valence of Nevada tell-us about its geography?

5

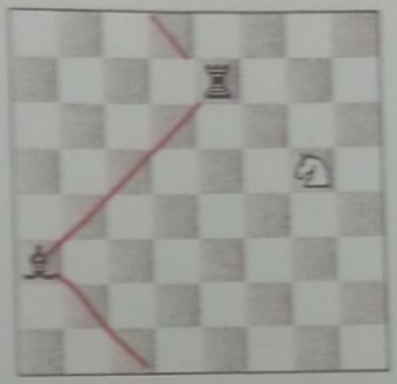
# of neighboring states is 5.

(c) Is it possible to start in some state, cross each border exactly once, and return home? If so, draw your path on the map. If not, convince me you're right.

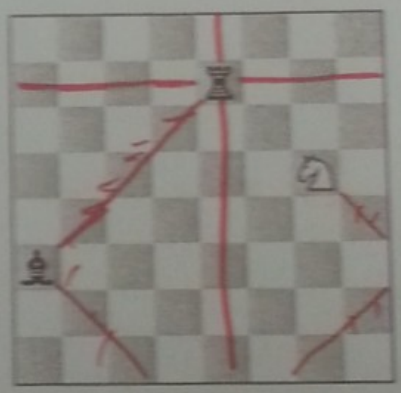
No. That would correspond to finding an Euler cycle in the graph, but that is impossible since we have a vertex with valence 5 (odd).

Problem 6 (5 points each). *Recall that in chess bishops travel diagonally, and the rooks travel up/down or left/right, as far as they want. In the pictures below, the bishop is on the left and the rook is in the middle.*

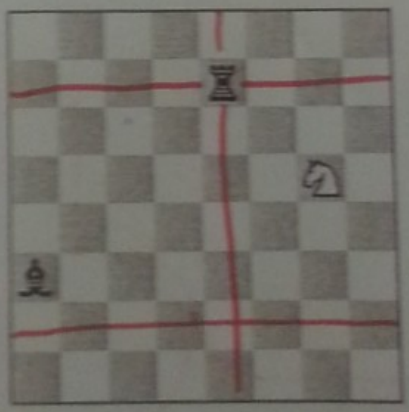
(a) If we glue just the top and bottom of the chess board, indicate all the places the bishop can reach in one move.



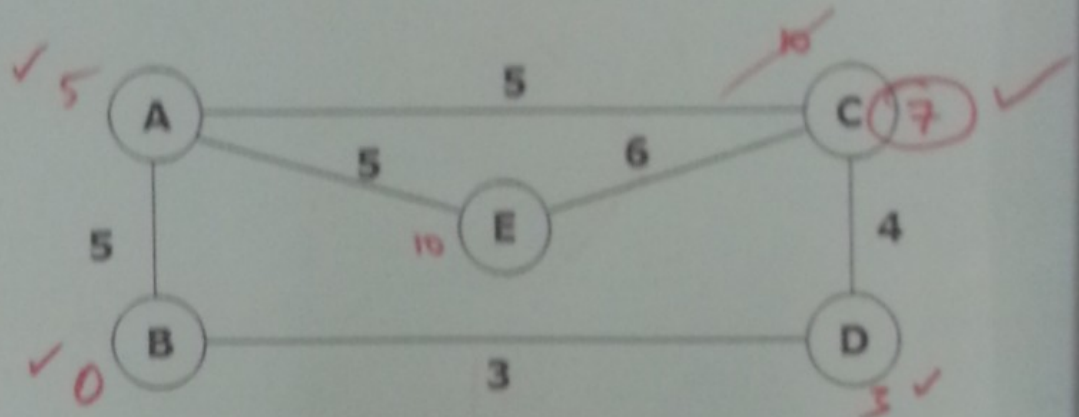
(b) If we glue just the left and right sides of the chess board, indicate all the places the rook can reach in one move.



(c) If we glue just the left and right sides of the chess board, but with a twist, indicate all the places the rook can reach in one move.



Problem 7 (5 points). *Use Dijkstra's algorithm to find the distance from B to C. Make sure to show all work and tell me the answer.*



distance is 7.

Problem 8 (5 points). *Suppose you are inside the following curve and have to stay one unit away from the boundary. Shade in all the places you could be.*