

8 pages. 15 problems. 160 points. No calculators. Show all work.

Problem 1 (5 points each). Consider the sets $A = \{\{\emptyset\}, \text{dog}, \text{cup}, \emptyset\}$, $B = \{\text{book}, \text{phone}, \text{dog}\}$, $C = \{\{\text{cat}\}\}$.

(a) How many elements does A have? List them without repetition.

(b) Is $\{\text{cat}\}$ an element of C ? Convince me.

(c) Is \emptyset an element of B ? Convince me.

Problem 2 (5 points). You're choosing a car to buy, and you expect to keep it for 10 years. The first car costs \$22,000 and has a 2% chance of needing a serious repair during 10 years. The second car costs \$20,000 and has a 50% chance of needing a serious repair during 10 years. If a serious repair costs \$5,000, which car should you get? (Hint: what is the expected cost of ownership?)

Problem 3 (5 points). You shuffle a deck of cards and flip a coin. What is the probability that the top card is an ace and the coin is not heads?

Problem 4 (5 points each). *Suppose you work at a bar and have to check a customer's ID if the customer looks under 35 (call this a "positive"). For older customers (over 35), your decision to check the ID is correct 98% of the time, and for younger customers (under 35), your decision is correct 99% of the time. Suppose you work with 5000 customers in one year, of which 1000 are under 35.*

(a) *What is the false positive rate for your decision (as a percentage)?*

(b) *How many younger customers do you miss?*

(c) *You decide to check a customer's ID. What is the probability the customer is over 35?*

Problem 5 (5 points each). *You have just been appointed vice president of the Chicago school system, and are asked to figure out what factors make a school good. Briefly describe how each type of study might work, and what the drawbacks would be.*

(a) *Interactive/opinion-based*

(b) *Correlational*

(c) *Experimental*

Problem 9 (5 points each).

(a) Convert 74_{10} to base-8.

(b) Convert 74_8 to base-10.

Problem 10 (5 points each).

(a) Write out the truth table for the expression “A and (B or C)”.

(b) Does it have the same meaning as “(A and B) or C”?

Problem 11 (5 points each). Consider the phrase $A = \text{“I like cookies and brownies, but not donuts.”}$

(a) What are the basic logical pieces of A ?

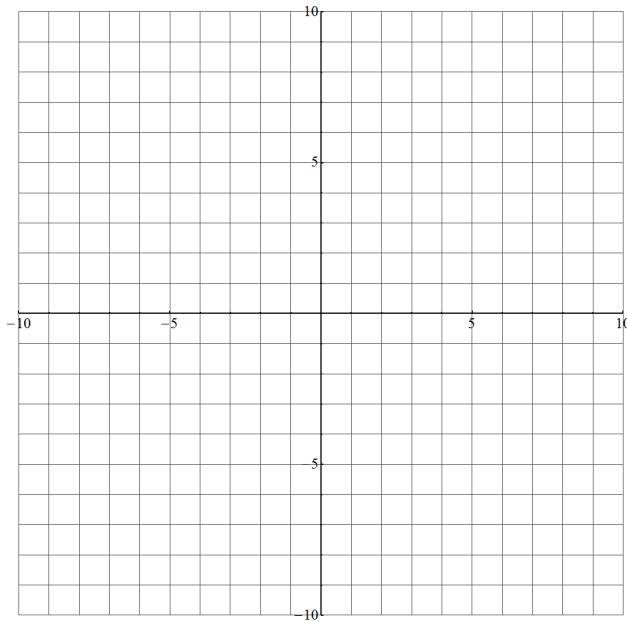
(b) Write A as a boolean expression using the basic logical pieces.

(c) Negate the boolean expression, making sure to simplify fully.

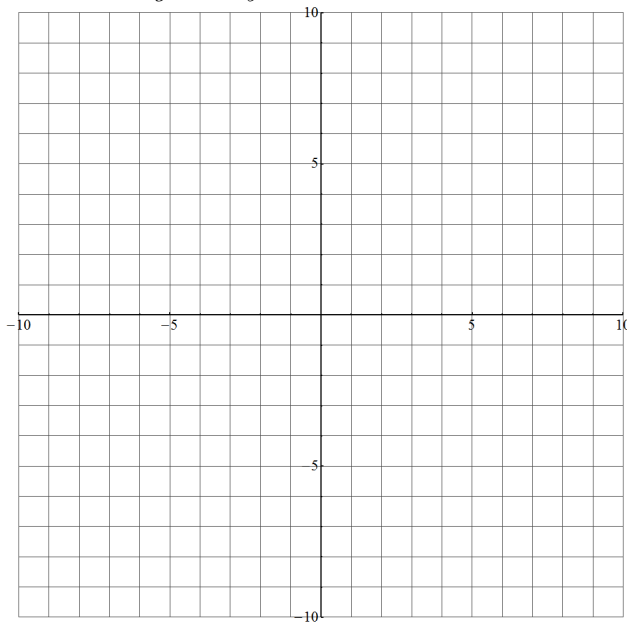
(d) Write out “not A ” in plain English, using your previous answers. (Hint: double-check that your answer is actually the opposite of A .)

Problem 12 (5 points each). *In each geometry, draw a geodesic between the points $A = (6, 3)$ and $B = (2, -2)$, and find the distance between the points.*

(a) *Paris geometry*



(b) *Polish comb geometry*

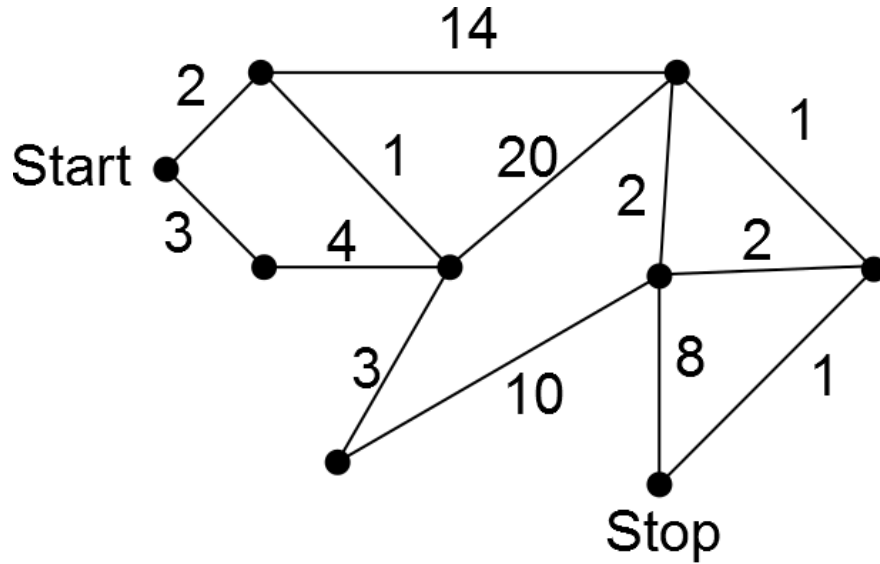


Problem 13 (5 points each). Think of the DC metro map as a graph.



- (a) What are the vertices of the graph?
- (b) What are the edges of the graph?
- (c) What is the valence of most points?
- (d) Suppose the Metro Center station is closed to all traffic. Is the graph still connected?

Problem 14 (10 points). Use Dijkstra's algorithm to find the distance from Start to Stop. Make sure to show your work and tell me the answer.



Problem 15 (5 points). Suppose you are inside the following curve and have to stay one unit away from the boundary. Shade in all the places you could be.

