

Math 675 Homework 11

Due 11/28/2018

1. Let V be a normed real vector space and $F : V \rightarrow \mathbb{R}$ a linear functional. Prove that F is continuous if and only if $N(F)$ is closed. (Hint: The forward direction requires no work at all; for the backward direction, assume F is not bounded and show that $N(F)$ is not closed by perturbing a non-zero point.)
2. Prove that each of the following functionals is continuous on $C_\infty[0, 1]$ and compute its norm:
 - (a) $f(x) = ax(0) + bx(1)$,
 - (b) $g(x) = \int_0^{1/2} x(t)dt - \int_{1/2}^1 x(t)dt$
3. Prove that if $p < q$ and f is a linear functional on $C_p[a, b]$ then it is also continuous on $C_q[a, b]$.
4. Give an example (with justification) of a linear functionals f and g on $C[a, b]$ such that:
 - (a) f is continuous with respect to d_∞ but not d_1 .
 - (b) g is continuous with respect to d_2 but not d_1 .
5. Let V_0 be a normed real vector space and V its completion. Prove that V_0^* and V^* are isomorphic Banach spaces.