Math 675 Homework 5

Due 9/26/2018

Warning: these problems are more tedious than hard, and mostly come down to unwrapping all of the definitions.

1. Suppose two metric spaces X and Y are bi-Lipschitz equivalent. Prove that that X is complete if and only if Y is complete.
2. Give an example of homeomorphic metric spaces X and Y such that X is complete but Y isn't. (Hint: arctan.)
3. Following Example 5 in Section 7.1, prove that ℓ_∞ is complete.
4. Let X be a metric space, and Y the completion of X defined in class. Let $\{y_i\}$ be a sequence of points of Y, and for each i let $\{x_i^j\}_{j=1}^\infty$ be a Cauchy sequence representing.
   (i) Prove that the diagonal sequence $z_i = x_i^i$ is Cauchy.
   (ii) Let y be the limit point of z_i. Prove that the sequence y_i converges to y.