Math 675 Homework 6

Due 10/3/2018

1. Give an example of a contraction that does not have a fixed point.
2. The contraction theorem requires a mapping that is L-Lipschitz for $L < 1$. Prove that the result is sharp. That is, allowing $L = 1$ in the theorem would make it false.
3. Prove that a closed subset of a complete space is complete.
4. Use the Picard Theorem to find a few approximate solutions to the differential equation $\frac{dy}{dx} = y$, with $y_0 = 1$, $x_0 = 0$, and initial guess of $\phi_0(x) = 0$. Then write down the solution as a series.
5. Prove that a closed subset of a compact space is compact. (Hint: its complement is an open set.)
6. Let $A, B \subset X$ be two compact subsets of a metric space X. Prove that $A \cup B$ and $A \cap B$ are both compact.