Math 675 Homework 8

Due 10/31/2018

1. Write $(1, 2, 3)$ as a linear combination of the vectors $(1, 1, 1)$, $(3, 5, 6)$ and $(7, 8, 9)$ using orthonormalization. Provide rounded numerical calculations, rather than exact ones.

   Proof. Let $u_1 = (1, 1, 1)$, $u_2 = (3, 5, 6)$ and $u_3 = (7, 8, 9)$. Then orthogonalizing gives

   $$v_1 = 0.58u_1 = (0.58, 0.58, 0.58)$$

   $$\begin{aligned} v_2' &= u_2 - (u_2, v_1)v_1 \\ &= u_2 - 8.08v_1 = (-1.67, 0.33, 1.33) \end{aligned}$$

   $$v_2 = v_2'/2.16 = (-0.77, 0.15, 0.62)$$

   $$\begin{aligned} v_3' &= u_3 - (u_3, v_1)v_1 - (u_3, v_2)v_2 \\ &= u_3 - 13.86v_1 - 1.39v_2 = (0.07, -0.21, 0.14) \end{aligned}$$

   $$v_3 = v_3'/0.27 = (0.27, -0.80, 0.53)$$

   We can then write

   $$\begin{aligned} (1, 2, 3) &= \langle(1, 2, 3), v_1\rangle v_1 + \langle(1, 2, 3), v_2\rangle v_2 + \langle(1, 2, 3), v_3\rangle v_3 \\ &= 3.46v_1 + 1.39v_2 + 0.27v_3 \end{aligned}$$

   Plugging in the formulas for v_1, v_2, v_3 in terms of u_1, u_2, u_3 we get:

   $$\begin{aligned} (1, 2, 3) &= 3.46(u_1/.58) + 1.39(2.16(u_2 - 8.08v_1)) + 0.27(0.27(u_3 - 13.86v_1 - 1.39v_2)) \\ &= 5.67u_1 + 1.39(2.16u_2 - 17.45v_1) + 0.7u_3 - 1.01v_1 - 0.10v_2 \end{aligned}$$

   □

2. Let $\|\cdot\|$ be the norm in $\mathbb{R}^2$ for which the unit circle is a regular hexagon with side length 1. Prove that $\|\cdot\|$ is not induced by an inner product.
3. Let $f_i(x) = x^i$, $i \in \mathbb{N}_{\geq 0}$, be the basis of monomials in $C_2[a, b]$.
   (a) Is it true that every continuous function on $C_2[a, b]$ is of the form $\sum_{i=0}^{\infty} a_i f_i$? (Hint: take a derivative.)
   (b) Let g_i be the associated orthonormal basis. Is it true that every continuous function on $C_2[a, b]$ is of the form $\sum_{i=0}^{\infty} a_i g_i$?
   (c) Why don't the two results contradict each other?